# Explaining classifiers and data via propositional logic 

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## Overview

- Background
- Explaining classifiers with propositional logic
- Explaining data with propositional logic

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## Explainability and interpretability in machine learning



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- A practical challenge is that these classifiers are often black boxes.
- Essentially two options for overcoming these challenges: develop methods for either explaining classifiers or for producing classifiers that are inherently interpretable.


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- Formulas of $\operatorname{PL}[\Phi]$ are evaluated on $\Phi$-valuations $v$, i.e., mappings $v: \Phi \rightarrow\{0,1\}$.
- Formulas $\sim$ classifiers and valuations $\sim$ data points.


## Explaining classifiers: motivating examples

## Example

(1) Consider the following valuation $v:\{p, q, r\} \rightarrow\{0,1\}:$

$$
v(p)=1, v(q)=1, v(r)=0
$$

Let $\varphi:=p \wedge(q \vee \neg r)$. Clearly $v(\varphi)=1$, i.e., $\varphi$ accepts the valuation $v$. But why? One possible explanation for this is to note that $\varphi$ accepts any valuation which maps $p$ and $q$ to one.

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(-) On the other hand, consider the following valuation $v:\{p, q, r\} \rightarrow\{0,1\}$ :

$$
v(p)=1, v(q)=0, v(r)=1
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If $\varphi$ is the same formula as above, then $v(\varphi)=0$. This time this could be explained by observing that $\varphi$ rejects any assignment that maps $q$ to zero and $r$ to one.

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$$
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Otherwise the answer is No.

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(- Consider the following valuation $v:\{p, q, r\} \rightarrow\{0,1\}$ :

$$
v(p)=1, v(q)=0, v(r)=1
$$

Let $\varphi$ be as above. On input $(v, \varphi, 0,4)$ the answer is Yes, as witnessed by $\neg q \wedge r$.

## Useful observation

## Lemma (Jaakkola et al., RCRA 2022)

Let $(v, \varphi, b, k)$ be an input to the special explainability problem. Let $\psi_{v}$ denote the conjunction that corresponds to $v$ :

$$
\bigwedge_{v(p)=1} p \wedge \bigwedge_{v(p)=0} \neg p
$$

Now, if there is an explanation of size at most $k$ for $v(\varphi)=b$, then there is one which is essentially a subconjunction of $\psi_{v}$.

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For monotone formulas the special explainability problem is NP-complete. In fact, the lower bound holds already for monotone CNF-formulas.

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## Proof.

Lower bound from the dominating set problem. Let $G=(V, E)$ be a graph. For each $x \in V$ we associate a propositional symbol $p_{x}$. Consider now the formula

$$
\varphi:=\bigwedge_{x \in V}\left(p_{x} \vee \bigvee_{(x, y) \in E} p_{y}\right)
$$

and set $v\left(p_{x}\right)=1$, for every $x \in V$. Now, on input $(v, \varphi, 1,2 k-1)$ the answer is Yes iff $G$ has a dominating set of size at most $k$.

## Special explainability problem for decision trees



## Special explainability problem for decision trees



- In [Barceló et al., NeurIPS 2020] it was essentially established that the special explainability problem for Boolean decision trees is NP-complete. Lower bound is again from the dominating set problem, but the reduction is quite fancy.


## Special explainability problem for full PL

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The special explainability problem for PL is $\Sigma_{2}^{p}$-complete.

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## Proof.

For the lower bound a reduction from $\Sigma_{2} \mathrm{SAT}$. Fix a quantified Boolean formula

$$
\exists p_{1} \ldots \exists p_{n} \forall q_{1} \ldots \forall q_{m} \theta
$$

and consider the following propositional formula:

$$
\varphi:=\bigwedge_{i=1}^{n}\left(p_{i} \vee \bar{p}_{i}\right) \wedge\left(\bigvee_{i=1}^{n}\left(p_{i} \wedge \bar{p}_{i}\right) \vee \theta\right)
$$

Set $v$ to be a valuation which maps all the propositional symbols to one. Now, on input $(v, \varphi, 1,2 n-1)$ the answer is Yes iff the original instance of $\Sigma_{2}$ SAT is true.

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- Can be proved via a reduction from the shortest implicant problem: on input $(\pi, \varphi, k)$, where
(1) $\varphi$ is a DNF-formula
(2) $\pi$ is a conjunction of literals and $k \in \mathbb{N}$,
decide whether there exists a subconjunction $\pi^{\prime} \subseteq \pi$ with at most $k$ literals such that $\pi^{\prime} \models \varphi$. Proved in [Umans, 2001] to be $\Sigma_{2}^{p}$-complete.


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decide whether there exists a subconjunction $\pi^{\prime} \subseteq \pi$ with at most $k$ literals such that $\pi^{\prime} \models \varphi$. Proved in [Umans, 2001] to be $\Sigma_{2}^{p}$-complete.
- To get the $\Sigma_{2}^{p}$-hardness for DNF-formulas, one essentially needs to show that the shortest implicant problem remains $\Sigma_{2}^{p}$-hard even if $\pi$ is a maximal conjunction of literals.


## Special cases of the special explainability problem

- For CNF-formulas the validity problem is solvable in polynomial time, because we can just check whether all of the clauses contain $p$ and $\neg p$. Using this, given a conjunction $\chi$ and a CNF-formula $\varphi$ we can also check $\chi \models \varphi$ in polynomial time.


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## Corollary (Jaakkola et al., RCRA 2022)

Restrict attention to those instances of the special explainability problem where $b=0$. Then the problem is NP-complete for DNF-formulas.

## Application: How hard it is to explain a random forest?



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- A very popular machine learning model (e.g., easy to train).
- Caveat: much harder to interpretate than decision trees.


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## Proof.

A reduction from the case of DNF-formulas. Let $\varphi:=t_{1} \vee \cdots \vee t_{m}$ be an arbitrary DNF-formula. Each $t_{i}$ is just a conjunction of literals and hence can be implemented via a linear size decision tree $T_{i}$. Now, the following is a random forest that is equivalent with $\varphi$ :

$$
F:=\{T_{1}, \ldots, T_{m}, \underbrace{\top, \ldots, \top}_{m \text {-times }}\}
$$

Thus we can reduce efficiently the SE problem of DNF formulas to that of random forests.

## Related work: PI-explanations

- A conjunction $\pi$ of literals is a prime implicant of an another formula $\varphi$, if $\pi \models \varphi$ and for every $\pi^{\prime} \subset \pi$ we have that $\pi^{\prime} \not \models \varphi$.


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- Lot of recent implementation work on computing prime implicants for Boolean classifiers, see e.g. [Darwiche, LICS 2023] for references. The related algorithms are not trying to find prime implicants that are globally optimal.


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- Lot of recent implementation work on computing prime implicants for Boolean classifiers, see e.g. [Darwiche, LICS 2023] for references. The related algorithms are not trying to find prime implicants that are globally optimal.
- Related literature also contains results on the computational complexity of determining whether a given conjunction is a prime implicant. E.g. for arbitrary formulas of propositional logic it is $\mathrm{D}^{\mathrm{P}}$-complete.


## Explaining data: motivating example

## Example

Consider the following (very small) Boolean data set over a vocabulary $\left\{p_{1}, p_{2}, p_{3}, q\right\}$ :

| $p_{1}$ | $p_{2}$ | $p_{3}$ | $q$ |
| :---: | :---: | :---: | :---: |
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Using the symbols $p_{1}, p_{2}, p_{3}$ we would like to predict the value of $q$. Based on this data, one formula that seems to work well is ( $p_{1} \wedge p_{2}$ ).

## Theoretical and empirical error

- Fix a vocabulary $\Phi=\left\{p_{1}, \ldots, p_{k}\right\}$ and $q \notin \Phi$. Set $\Phi_{q}:=\Phi \cup\{q\}$. We assume an underlying but unknown probability distribution

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\mu:\left\{\Phi_{q} \text {-assignments }\right\} \rightarrow[0,1]
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- Goal is to find a formula $\varphi$ of $\operatorname{PL}[\Phi]$ which minimizes the theoretical (ideal) error:

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- Since $\mu$ is unknown, we can not calculate $\operatorname{err}_{\mu}(\varphi)$ directly. Instead, we approximate it by taking samples $S$ (= data sets) from $\mu$. We then evaluate formulas based on their empirical error:

$$
\operatorname{err}_{S}(\varphi):=\frac{1}{|S|} \sum_{\substack{v \in S \\ v(\varphi) \neq v(q)}} 1
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- Intuitively, a formula overfits to a sample if it focuses too much on the particular features of that sample.


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## Theorem (Uniform convergence for PL)

Let $k \in \mathbb{Z}_{+}$and $\delta>0$. If a sample $S$ of size at least

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\frac{1}{2 \varepsilon^{2}}(5 k \log (k)+\ln (2 / \delta))
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- Most of the time these type of bounds are not good enough to be practically relevant.


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- Each data set contained several non-Boolean attributes, so Booleanization was needed.


## Empirical results

- The first data set was the Statlog-German credit data set, classifies persons based on whether or not it is "risky" to give them a loan.
- 1000 data points and 68 attributes.
- The following formula of size six
$\neg($ negative_balance $\wedge$ above_median_loan_duration)
$\vee$ employment_on_managerial_level
had 0.27 as its test error.
- Naive Bayes classifiers had test error 0.25 and neural networks 0.24 .



## Empirical results

- The second data set was Breast cancer Wisconsin data set, classifies tumors based on whether or not they are benign.
- 683 data points ja 9 attributes.
- The following formula of size eight

$$
\neg(((p \wedge q) \vee r) \wedge s)
$$

had 0.047 as its test error.

- Naive Bayes classifiers had test error 0.026 and neural networks 0.032.



## Empirical results

- The final data set was the lonosphere data set, classifies radar signals based on whether or not they are "good".
- 351 data points and 34 attributes.
- The following formula of size seven

$$
((p \wedge q) \vee r) \wedge s
$$

had 0.14 as its test error.

- Naive Bayes classifiers had test error 0.1 and neural networks 0.04 .


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- In [Rudin, 2019] it is argued - based on empirical evidence - that for most problems with "meaningful structured covariates" different machine learning algorithms tend to perform similarly.


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Stop Explaining Black Box Machine Learning Models for High Stakes Decisions and Use Interpretable Models Instead

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- In particular, we might expect that interpretable models achieve similar accuracies as black box models (such as large neural networks). Our empirical results certainly support this claim.


## When do we need black box models?

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## Thanks! :)

