Formulas of propositional logic as interpretable classifiers

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Joint work with Tomi Janhunen, Antti Kuusisto, Masood Feyzbakhsh Rankooh and Miikka Vilander.

Propositional logic recap

- Fix a set $\tau := \{p_1, \ldots, p_d\}$ of propositional variables. The set of formulas of $\mathrm{PL}[\tau]$ is generated via the following grammar:

$$\varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi).$$

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- Given an **assignment**

$$s : \tau \to \{0, 1\},$$

  we define recursively $s(\varphi)$ as follows:

  - $s(\neg\varphi) = 1$ iff $s(\varphi) = 0$.
  - $s(\varphi \wedge \psi) = 1$ iff $s(\varphi) = 1$ and $s(\psi) = 1$.
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- The **size** of a formula φ is defined recursively as follows:

  - $\mathrm{size}(p) = 1$
  - $\mathrm{size}(\neg\varphi) = 1 + \mathrm{size}(\varphi)$
  - $\mathrm{size}(\varphi \wedge \psi) = \mathrm{size}(\varphi \vee \psi) = 1 + \mathrm{size}(\varphi) + \mathrm{size}(\psi)$

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- As an example, consider the following Boolean data set.

p_1	p_2	p_3	q
1	1	1	1
1	1	0	1
1	0	0	0
0	1	0	0

Based on this data, the formula $(p_1 \wedge p_2)$ seems to predict quite well the value of q.

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- **Goal** is to learn a propositional formula φ over τ with small **error**:
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- Since μ is unknown, this is done using samples $S \sim \mu^n$ and by minimizing **empirical error**:
$$\mathrm{err}_S(\varphi) := \frac{1}{n} \sum_{\substack{s \in S \\ s(\varphi) \neq s(q)}} 1$$

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  Works well **if** q is assumed to be equivalent to a conjunction of symbols from τ.
- Can be also made to work if q is equivalent to a conjunction plus "noise".

Learning general propositional formulas is in general hard

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- Restricting attention to formulas of $\mathrm{PL}[\tau]$ which are of polynomial size w.r.t. $|\tau|$ seems to make the problem feasible. Indeed, the VC-dimension of this class is only polynomial w.r.t. $|\tau|$. Thus, in principle, one needs only polynomially many samples.

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- However, [Kearns and Valiant, 1994] established that — under a standard cryptographic assumption — polynomial size propositional formulas can not be learned in polynomial time.

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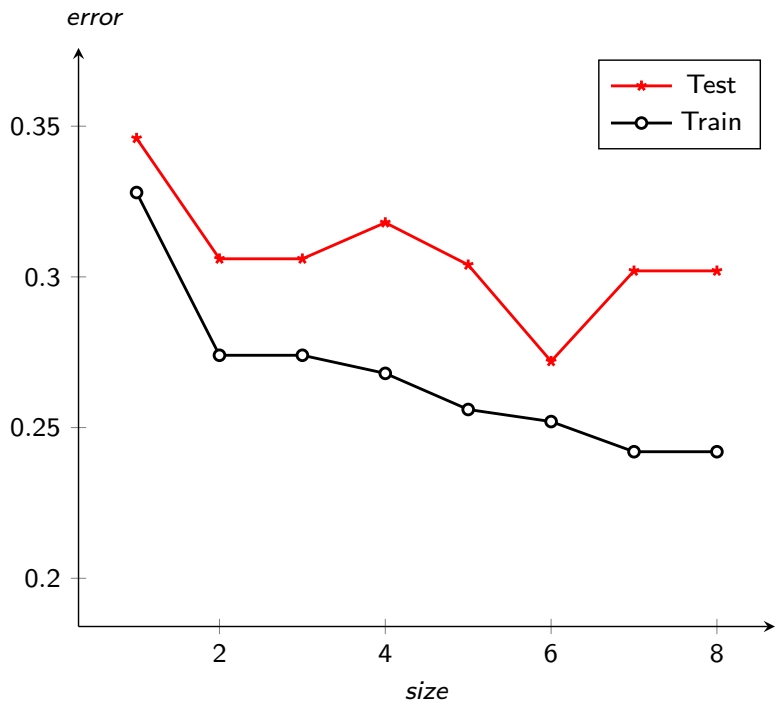
- Hope was that it would be sufficient to consider small values of k.
- Most real-world data sets are not Boolean, so one needs to first booleanize them.
  - Categorical features can be one-hot encoded.
  - For continuous features we simply split them at the median.

Empirical results from [Jaakkola et al., 2023]

- The first data set was the **Statlog-German credit** data set, classifies persons based on whether or not it is "risky" to give them a loan.
- 1000 data points and 68 attributes. The data was split 50-50 to training and testing sets.
- The following formula of size six

  $$\neg\,(\text{negative\_balance} \wedge \text{above\_median\_loan\_duration}) \vee \text{employment\_on\_managerial\_level}$$

  had 73% accuracy on the test set.
- In the literature e.g. neural networks had obtained 76% test accuracy.



Empirical results from [Jaakkola et al., 2023]

- The second data set was **Breast cancer Wisconsin** data set, classifies tumors based on whether or not they are benign.
- 683 data points ja 9 attributes. The data was split 50-50 to training and testing sets.
- The following formula of size eight

$$\neg(((p \wedge q) \vee r) \wedge s)$$

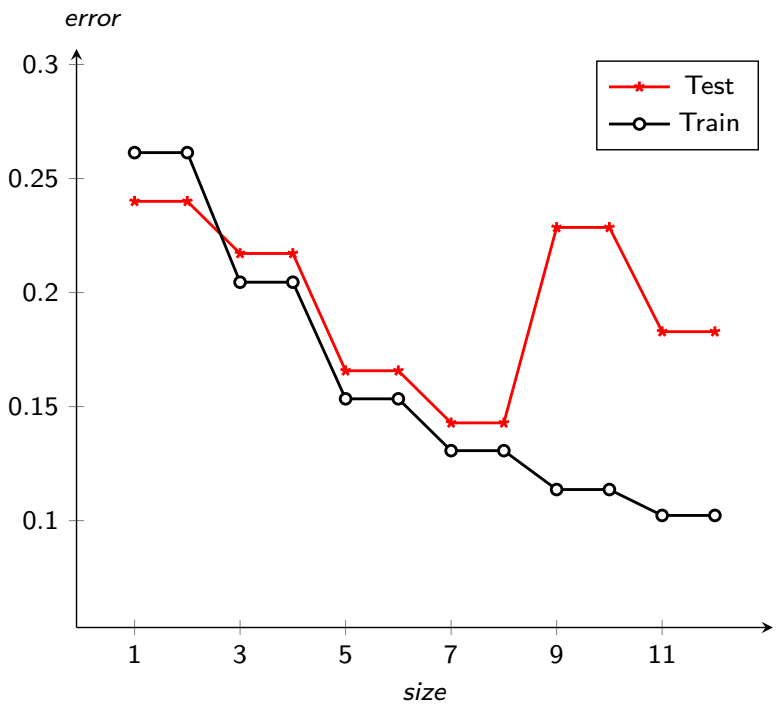
  had 95.3% accuracy on the test set.
- In the literature e.g. naive Bayes classifiers had obtained 97.4% test accuracy.

Empirical results from [Jaakkola et al., 2023]

- The final data set was the **Ionosphere** data set, classifies radar signals based on whether or not they are "good".
- 351 data points and 34 attributes. The data was split 50-50 to training and testing sets.
- The following formula of size seven

$$((p \wedge q) \vee r) \wedge s$$

  had 86% accuracy on the test set.
- In the literature e.g. neural networks had obtained 96% test accuracy.



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- Crude discretization already works quite well.
- Already short and interpretable formulas obtain good accuracies.

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- In fact, already [Holte, 1993] observed that for several real-world datasets one can get pretty good accuracies by using a single feature (comparable to ones obtained using decision trees).
- In [Jaakkola et al., 2024] we investigated an approach based on feature selection in more detail.

The approach of [Jaakkola et al., 2024]

- Given a sample S of assignments over $\tau \cup \{q\}$, we first select a **small** set $\sigma \subseteq \tau$ of "promising" features. We then compute a DNF-formula over σ which has a minimal empirical error w.r.t. S.

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- Given $t : \sigma \to \{0, 1\}$, we let

$$\Pr_S[q|t] := \frac{\text{number of assignments } s \text{ in } S \text{ for which } s \restriction \sigma = t \text{ and } s(q) = 1}{\text{number of assignments } s \text{ in } S \text{ for with } s \restriction \sigma = t}$$

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The following DNF-formula minimizes empirical error w.r.t. S

$$\bigvee \left\{ t \mid \Pr_S[q|t] > 1/2 \right\}.$$

Calculating $\Pr_S[q|t]$ from S is trivial.

Related approach

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  else if (sex is male) **and** (juvenile crimes$>$ 0) **then predict** `yes`

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- Since rule lists are optimized w.r.t. their size, the approach is also similar in spirit to that of [Jaakkola et al., 2023].

Some experiments using the approach of [Jaakkola et al., 2024]

Data set	Our method	Random forests	XGBoost
BankMarketing	89.5%	89.7%	89.6%
BreastCancer	95.3%	97.1%	96.4%
CongressionalVoting	95.6%	96.3%	96.3%
GermanCredit	71.5%	76.2%	72%
HeartDisease	82.8%	84.2%	81.2%
Hepatitis	85.0%	83.6%	79.7%
StudentDropout	81.0%	87.3%	87.3%

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- For our method we treated the number of used features as a hyperparameter. 10% of the training set was used as a validation set.
- For feature selection we tested F-test, χ^2-test and mutual information.

Some experiments using the approach of [Jaakkola et al., 2024]

- Results from Leave-One-Out Cross-Validation (LOOCV) for two high-dimensional data sets. Features related to gene expression profiles.

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- Results from Leave-One-Out Cross-Validation (LOOCV) for two high-dimensional data sets. Features related to gene expression profiles.
- Colon, 63 data points and 2000 features. All features were three-valued.
  1. [Jaakkola et al., 2024] with F-test and χ^2-test: 82.3% accuracy.
  2. Random forest with 1000 decision trees: 82.3% accuracy.
  3. Logistic regression with ℓ_1-regularization: 77.4% accuracy.
  4. Support vector machine with linear kernel: 85.5% accuracy.

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- Leukemia, 73 data points and 7070 features. All features were three-valued.
  1. [Jaakkola et al., 2024] with F-test and χ^2-test: 97.2% and 81.9% accuracies respectively.
  2. Random forest with 1000 decision trees: 98.6% accuracy.
  3. Logistic regression with ℓ_1-regularization: 95.8% accuracy.
  4. Support vector machine with linear kernel: 98.6% accuracy.

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  - E.g. with a 80-20 split a decision tree of depth 10 obtained a test accuracy of 81.4% while a decision tree of depth 20 obtained a test accuracy of 91.9%.
- Even if our method does not produce a good **classifier**, it is still able to identify interesting properties from the data set.

Bonus: estimating best possible accuracy

- Given a probability distribution, we might be interested in estimating

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$$\mathrm{err}(S) := \mathsf{min}\{\mathrm{err}_S(\varphi) \mid \varphi \in \mathrm{PL}[\tau]\}.$$

- We have $\mathbb{E}(\mathrm{err}(S)) \leq \mathrm{err}(\mu)$. Unfortunately $\mathrm{err}(S)$ is not an unbiased estimator of $\mathrm{err}(\mu)$.

Bonus: estimating best possible accuracy

Theorem ([Jaakkola et al., 2023])

Suppose that $S \sim \mu^n$. *Then*

$$\mathbb{E}[\mathrm{err}(S)] \geq \mathrm{err}(\mu) - \frac{1}{\sqrt{n}} \sum_{t:\tau \to \{0,1\}} \sqrt{\mu(q|t)(1-\mu(q|t))\mu(t)}$$

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- We have the bound

$$\frac{1}{\sqrt{n}} \sum_{t:\tau \to \{0,1\}} \sqrt{\mu(q|t)(1-\mu(q|t))\mu(t)} \leq \frac{1}{2}\sqrt{\frac{2^{|\tau|}}{n}}.$$

  Very pessimistic: matching lower bound is realized by setting μ to be the uniform distribution over $\tau \cup \{q\}$.

Thanks!

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